Some Solutions to ***Finite-Dimensional Vector Spaces***

by Paul R. Halmos

Solutions by Chad A.
ionjump@gmail.com

As I am working through this book, I am writing my solutions in a notebook. I have typed a few solutions here. If there is interest, I will type more of my solutions, maybe even all of my solutions, so please email me to let me know.

Section 36

If a transformation A has these properties:

(i). If $x_1 \neq x_2$ then $\mathrm{Ax}_1 \neq \mathrm{Ax}_2$

(ii). To every vector y ther corresponds (at least) one vector x such that Ax=y

then we shall say that A is invertible.

1. a. The complex conjugate is inverable

   b. If $x_1(t) = 1$ and $x_2(t) = 2$ then $x_1 \neq x_2$

   but

   $$\mathrm{Ax}_1 = x_1(t+1) - x_1(t) = 1 - 1 = 0$$

   and

   $$\mathrm{Ax}_2 = x_2(t+1) - x_2(t) = 2 - 2 = 0$$

   so $\mathrm{Ax}_1 = \mathrm{Ax}_2$ so the first property of invertable linear transformations is not valid so this linear transformation is not invertable.

   c. V is the k-fold tensor product of a vector space with itself; A is such that

   $$A(x_1 \otimes \cdots \otimes x_k) = x_{\pi(1)} \otimes \cdots \otimes x_{\pi(k)}$$

   where π is a permutation of $\{1, \ldots, k\}$. Clearly, the reverse permutation will get you the original tensor product. If $\mathrm{Ax}_1 = \mathrm{Ax}_2$ then $x_1 = x_2$ because they will be the same tensor product so (i) is true. (ii) is clearly true.

2.

3.

4.

5.

6.

7.

8.

9.

10. Same as the definition for the invertibility within a single vector space except for x is in one vector space and y and Ax are in the other vector space.

    a) Not inverible. In converting a vector to a scalar, information is lost. (i) will not be valid.

    b) Not invertible. If x_1 and x_2 have the same first coordinate but different second coodinate then $x_1 \neq x_2$ but $\mathrm{Ax}_1 = \mathrm{Ax}_2$ so (i) is not valid.

    c) Not invertible. Two different vectors in U can produce the same coset in V

    d) This is similar to the case of (a). Two different x can produce the same linear functional on V. The vector information is lost when x_0 is inserted into w and converted to a linear functional on V_0.